Some multivariate risk indicators; estimation and application to optimal reserve allocation

Véronique Maume-Deschamps, Peggy Cenac, Clémentine Prieur

Session : Algorithmes Stochastiques

Some multivariate risk indicators ; estimation and application to optimal reserve allocation

par Peggy Cénac, **Veronique Maume-Deschamps** et Clémentine Prieur

We consider a vectorial risk process :

$$X_i = \begin{pmatrix} X_i^1 \\ \vdots \\ X_i^d \end{pmatrix}$$

X_i^k corresponds to the gains of the kth business line during the ith period : $X_i^k = G_i^k - L_i^k$ where G_i^k denotes the incomes and L_i^k denotes the losses. We are interested in the cumulative gain : $Y_i^k = \displaystyle\sum_{p=1}^{i} X_p^k$. For a given period i, the variables $(X_i^k)_{k=1,\ldots,d}$ may be dependent. Some temporal dependence (with respect to index i) may also be taken into account. Given an initial capital u we assume that it is allocated to each line of business. Let u_i denotes the initial capital of the ith line of business, $u_1 + \cdots + u_d = u$. We aim to optimize the capital allocation with respect to some risk indicator. We introduce new risks indicators, considering that the main risk drivers for the overall company have been identified and the global solvency capital requirement has been computed. These new indicators reveal the marginal solvency capitals for each line of business. A way to avoid as far as possible that some lines of business become insolvent too often could be to minimize these risk indicators, under a fixed total capital constraint. These might be achieved if some capital fungibility between lines of business or between entities is possible. One possible way to define optimality of the global reserve allocation is to minimize the expected sum of the penalties that each line of business would have to pay due to its temporary potential insolvency. **A localized risk indicator.** A multivariate risk indicator that takes into account the dependence structure may be :

$$B(u_1, \ldots, u_d) = \sum_{k=1}^{d} \mathbb{E}\left(\sum_{p=1}^{n} \mathbf{1}_{\{R_p^k < 0\}} \mathbf{1}_{\{\sum_{k=1}^{d} R_p^k > 0\}} \right),$$

with $R_p^k = u_k + Y_p^k$. This risk indicator gives an indication on some average time to ruin, it has been introduced in Loisel 2004 and is not, in general, convex. We shall consider, given a differentiable and convex function $g_k : \mathbb{R} \to \mathbb{R}$ satisfying $g_k(x) \geq 0$ for $x \leq 0$, $k = 1, \ldots, d$:

$$I(u_1, \ldots, u_d) = \sum_{k=1}^{d} \mathbb{E}\left(\sum_{p=1}^{n} g_k(R_p^k) \mathbf{1}_{\{R_p^k < 0\}} \mathbf{1}_{\{\sum_{k=1}^{d} R_p^k > 0\}} \right).$$

The function g_k represents the cost that the kth business branch has to pay if it becomes insolvable. The problem is to find a minimum of I under the constraint $v_1 + \cdots + v_d = u$. Formally, we are looking for $u^\star$ such that

$$I(u^\star) = \inf_{v_1 + \ldots + v_d = u} I(v).$$

Unless for some very specific models, we are not able to compute explicitly $u^\star$ which realize the minimum. Thus, we propose to solve this minimization problem by using stochastic algorithms. We provide a proof of almost sure convergence of our algorithm as well as an estimation of the probability error. Then we perform some simulations.

Adresses :

Peggy Cénac
Institut de Mathématiques de Bourgogne

Veronique Maume-Deschamps
ISFA, Université de Lyon 1

Clémentine Prieur
Laboratoire Jean Kuntzmann, Grenoble